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On the mean path of buoyant, bent-over chimney plumes

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Careful plume-rise observations by photographic means have been made on smoke plumes from the Lakeview Generating Station (Ontario) and compared with some existing theoretical formulae. Supporting data in considerable detail on stack parameters were available.

A linearly rising portion of the plume was found beyond a specific transition point in a neutral atmosphere. Also, the previously established simple power law $z \sim F^{\frac{1}{3}} U^{-1} x^{\frac{2}{3}}$, was found to agree quite well with observation up to the transition point.

In an unstable atmosphere the plume was sometimes above and sometimes below a corresponding plume in neutral conditions, under the opposing influences of increased dilution and the direct effect of instability in promoting plume rise.

1. Introduction

In spite of the obvious practical importance of the buoyant rise of chimney effluents in connexion with air pollution problems, considerable confusion exists even as regards the gross features of hot plume behaviour. For instance, it is commonly taken for granted that a chimney plume levels off at an 'effective stack height', not only in a stable but also in a neutral or near-neutral atmosphere, in spite of the fact that theory and the scant available experimental evidence suggest otherwise. With an ostrich-like philosophy, the effective stack height is often defined to be the point where the smoke plume is just lost sight of. It is then not very surprising to find that the observed thermal rise of the plume depends, for example, on a power of the heat flux ranging from $\frac{1}{4}$ to 1·0, influenced by a number of factors including, presumably, the observer's eyesight.

Clearly, any further progress requires the collection of reliable experimental data on plume rise, particularly at relatively large distances from the source. Following previous theoretical and experimental studies (Csanady 1956, 1961, 1965) a systematic observational programme on the chimney-plume of a large power station near Toronto has been carried out, the results of which are reported herein. Data have been successfully collected on the rise of the plume under neutral as well as unstable conditions to considerable distances from the chimney. Great care has been exercised in collecting and reducing the data in order to avoid errors of perspective, etc., and it is felt that the results constitute a more reliable set of measurements than many published in the past.

2. Theoretical considerations

When the hot gases of an industrial chimney enter the atmosphere, they possess considerable buoyancy which rapidly increases their vertical momentum. At the same time, turbulent movements disperse their excess heat and momentum over the surrounding fluid, so that the plume visibly 'grows' by 'entrainment' of ambient air. In the understanding of the plume's behaviour this turbulent spreading mechanism is the critical element: the equations of motion cannot be written down unless some realistic model of turbulent mixing is adopted.

Physically, two fairly distinct 'sources' of turbulence exist: one, the plume's 'self-generated' turbulence due to the relative motion of the buoyant and non-buoyant fluid; two, the environmental turbulence naturally present in the wind (which, of course, is due to the relative motion of air and ground). Very often, in the early stages of plume motion the self-generated turbulence dominates the mixing process, while far downstream from the chimney the environmental turbulence is responsible for most of the spreading. Priestley (1956) appears to have been the first to identify these two important phases in the behaviour of a chimney plume.

Further reflexion shows that the environmental turbulence-dominated phase may again be subdivided into two sections (Csanady 1965): one, where the important diffusing eddies belong to the inertial subrange; two, where the plume diameter is of the order of the size of the energy containing eddies or larger. Inertial subrange dominated diffusion is known to be quite rapid and may be expected to lead to a relatively rapid 'levelling off' of the plume, while the contrary is the case in the 'final' phase where turbulent mixing resembles molecular diffusion. In summary, we may speak of three distinct phases of plume behaviour.

(1) 'Initial phase': self generated turbulence dominates mixing.

(2) 'Intermediate phase': environmental turbulence in the inertial subrange dominates mixing.

(3) 'Final phase': the energy containing eddies of environmental turbulence dominate mixing.

It is, of course, a considerable simplification to speak of three such distinct phases: in reality there is likely to be considerable overlap, e.g. between the initial and intermediate phases. Also, for a large enough initial plume diameter the 'intermediate' phase may be very short or even completely absent. Nevertheless, such a classification helps in the understanding of a rather complex problem. Visual evidence *does* suggest exactly such a three-phase model of plume behaviour (Csanady 1961). In the complete absence of a horizontal wind, of course, the plume is vertical and its turbulence is entirely self-generated; this, however, is a relatively rare event, and one is normally concerned with the 'bent over' plume at least when it comes to air pollution problems.

Quantitatively, turbulent mixing may be represented by either of two approximate methods:

(*a*) the simple and successful 'turbulent entrainment' hypothesis of Morton, Taylor & Turner (1956);

(*b*) the use of eddy viscosity and conductivity, whether in conjunction with 'mixing length' arguments, or not.

In order to illustrate in the simplest possible manner the implications of the two theoretical alternatives for the three different phases of plume behaviour, we shall assume that the plume axis is nearly horizontal. Let U be the (uniform) wind speed, $\Delta\rho$, w and R a characteristic density defect, vertical velocity and radius of the plume respectively. Then the analogues of the three equations of Morton *et al.* (1956), expressing conservation of mass, momentum and energy, are for a section of the nearly horizontal plume in a *neutral* atmosphere (assuming a 'top hat' profile of temperature and vertical velocity, i.e. constant w or $\Delta\rho$ from 0 to R):

$$U(dR^2/dx) = 2\alpha wR, \tag{1}$$

$$U[d(R^2w)/dx] = R^2g(\Delta\rho/\rho) = F/U, \tag{2}$$

$$R^2Ug(\Delta\rho/\rho) = F = \text{const.} \tag{3}$$

Here F is the 'flux of buoyancy' and α an entrainment constant such that αw is the 'influx velocity' of ambient fluid into the plume. This formulation holds for the 'initial' phase only, because the influx velocity is assumed to be due solely to the plume's own motion. The mean path $z(x)$ of the plume may be found from these equations by noting that

$$dz/dx = w/U. \tag{4}$$

For negligible initial radius R_0 and velocity w_0 the solution is

$$z/l = C(x/l)^{\frac{2}{3}}, \tag{5}$$

where $C = (3/2\alpha^2)^{\frac{1}{3}} = \text{const.}$, and $l = F/U^3$ is a 'length scale' of buoyant movements.

In many cases in practice one is concerned with plume behaviour at distances from the source of the order of hundreds of metres. On this scale the effects of initial plume size or efflux velocity are often negligible, so that equation (5) may be expected to apply beyond some not very significant distance from the source. Specifically, one may expect the effects of initial size to be negligible if

$$x \gg R_0, \tag{6}$$

and that of initial momentum if

$$g(\Delta\rho_0/\rho)(x/U) \gg w_0, \tag{6a}$$

i.e. at $x \gg w_0 U\rho/(g\Delta\rho_0)$, where $\Delta\rho_0$ is the initial density defect. The last inequality expresses the fact that at some large enough distance the buoyancy generated momentum dominates initial momentum. Normally these inequalities are satisfied for power station chimneys (to take one example) at distances of $x = 100$ m or larger. The approximate validity of equation (5) for plumes from large chimneys in the initial phase has been noted before (Scorer 1959; Csanady 1956, 1961). It should be borne in mind, however, that this conclusion is not valid for plumes only marginally buoyant.

In the 'intermediate' phase of plume behaviour the rate of growth of the plume is, by hypothesis, determined by the atmospheric eddies of the inertial subrange.

This accelerating growth is known to be described by the Richardson–Batchelor relationship (Batchelor 1950, 1952)

$$\tfrac{1}{2}(dR^2/dt) = a_1 \epsilon^{\frac{1}{3}} R^{\frac{4}{3}}, \tag{7}$$

where ϵ is the turbulent energy dissipation per unit mass and a_1 is a constant of order unity, differing from Batchelor's owing to the fact that R is the standard deviation multiplied by a non-unit factor. For a plume travelling along with a uniform wind of speed U, (7) may be written as

$$U(dR^2/dx) = 2a_1 \epsilon^{\frac{1}{3}} R^{\frac{4}{3}}. \tag{7a}$$

This relationship now replaces (1) and is easily interpreted physically: the 'influx velocity' αw of the initial phase gives way to an influx velocity $a_1 \epsilon^{\frac{1}{3}} R^{\frac{1}{3}}$, which is proportional to the 'velocity scale' of eddies of size R in the inertial subrange. Integration of the set of equations (2), (3) and (7 a) yields now

$$U\frac{dz}{dx} = w = \frac{R_1^2 w_1 + F(x-x_1)/U^2}{[R_1^{\frac{2}{3}} + (2a_1\epsilon^{\frac{1}{3}})(x-x_1)/3U]^3}, \tag{8}$$

where R_1 and w_1 are radius and vertical velocity of the plume at the beginning of the intermediate phase. At large $(x-x_1)$, if the intermediate-phase relationships still hold, this leads to

$$dz/dx = \text{const. } (x-x_1)^{-2}, \tag{8a}$$

so that the plume tends to a constant asymptote, which could then be regarded as an 'effective chimney height'.

In the *final* phase of plume behaviour the energy containing eddies determine the mixing and the rate of growth is known to be described by (Batchelor 1950, 1952)

$$\tfrac{1}{2}(dR^2/dt) = a_2 vL, \tag{9}$$

where v and L are R.M.S. turbulent-velocity and (diffusion) length scale respectively, the latter being in order of magnitude equal to the dissipation length scale, v^3/ϵ. The constant a_2 has to be introduced because R differs from the standard deviation by a multiplicative factor; since this factor is about 2, a crude estimate of a_2 is 4.

On comparing (9) with (1) we find that the influx velocity into the plume is now $a_2 v(L/R)$, i.e. decreasing with increasing plume size. Clearly by this time the 'turbulent entrainment' hypothesis has been considerably modified. Integration of the set of equations (2), (3) and (9) results in

$$w = \frac{R_2^2 w_2 + F(x-x_2)/U^2}{R_2^2 + (2a_2 vL)(x-x_2)/U}, \tag{10}$$

where R_2 and w_2 are radius and vertical velocity of the plume at the beginning of the final phase. At large $(x-x_2)$ this reduces to

$$dz/dx = F/2a_2 vLU^2 = \text{const.}$$
$$= \frac{1}{2a_2}\left(\frac{l}{L}\right)\left(\frac{U}{v}\right), \tag{10a}$$

so that the path of the plume is asymptotically a straight line of a slope given by (10*a*).

Turning now to the application of the eddy-viscosity and conductivity hypothesis to a nearly horizontal plume, the characteristic vertical velocity may at once be deduced from the differential equations (Csanady 1965) as

$$w = b(F/2KU), \tag{11}$$

where K is the eddy exchange coefficient, and b is a constant of order 10^{-1}, as calculated from the theoretical flow pattern. If the plume 'radius' is taken to be the standard deviation of dispersion (of heat or matter in the plume) then the eddy diffusivity is related to radius by

$$K = \tfrac{1}{2}U(dR^2/dx). \tag{12}$$

In the *initial* phase of plume behaviour the turbulent movements are due to the movement of the plume itself. Hence a realistic assumption for the diffusivity is

$$K = cwR, \tag{13}$$

where c is a constant, probably of order 10^{-1} (by analogy with other turbulent phenomena, turbulent velocities being normally one order of magnitude smaller than mean velocities). Substituting (13) into (11) and (12) one finds after some elementary manipulations, for zero initial radius and momentum,

$$z/l = C(x/l)^{\frac{2}{3}}, \tag{14}$$

where $C = (9b/8c^2)^{\frac{1}{3}}$ and $l = F/U^3$ is the length-scale of buoyant movements as defined before. The result is thus identical with that obtained on the basis of the 'turbulent entrainment' hypothesis (equation (5)), the constant C also having the same order of magnitude.

It may be pointed out here that if we were to write†

$$K = \text{const. } x \tag{15}$$

(as would be the case in the initial phase of diffusion from a point source if the environmental turbulence were dominant) the path would be given by $z \sim \ln x$, a relationship found in a similar analysis by Bosanquet, Carey & Halton (1950), which is known certainly not to apply in the initial phase.

In the inertial subrange-dominated intermediate phase the diffusivity may be written (Batchelor 1950)

$$K = \beta\epsilon^{\frac{1}{3}}R^{\frac{4}{3}}, \tag{16}$$

where β is a constant of order unity. On substitution into equations (11) and (12) one finds

$$U(dz/dx) = w = (bF/2U\beta\epsilon^{\frac{1}{3}})[R_1^{\frac{2}{3}} + (2\beta\epsilon^{\frac{1}{3}}/3U)(x-x_1)]^{-2}. \tag{17}$$

At large values of $(x-x_1)$ this gives $w \sim (x-x_1)^{-2}$, i.e. z again tending to an asymptotic height, exactly as found with the aid of the turbulent entrainment hypothesis.

† It has been pointed out by a referee that this equation does *not* apply to a jet, as erroneously stated in Csanady (1965).

In the *final* phase the eddy diffusivity is constant

$$K = vL, \tag{18}$$

where v and L are characteristic velocity and length scales of the energy-containing eddies. With (11) this gives at once

$$\frac{z}{l} = \frac{b(l/L)}{2(v/U)} \frac{x-x_2}{l}. \tag{19}$$

This is again a linear relationship, identical with (10a) except for a slight difference in value of the constant, caused by different assumptions regarding the velocity and density defect profiles. In practice this constant would have to be determined empirically.

Theory provides therefore the following guidelines:

(1) In the initial phase of plume behaviour, where the plume's self-generated turbulence is the dominant mixing agency, the plume path is approximated by the $z \sim x^{\frac{2}{3}}$ relationship. This result follows from either approximate theory ('turbulent entrainment' or 'eddy diffusivity') and has been confirmed experimentally in the past.

(2) In the intermediate phase, dominated by atmospheric eddies in the inertial subrange, the plume should have a strong tendency to level off. This is suggested by conclusions based on either approximate theory, although no observational evidence exists in its favour.

(3) In the final phase (for which no observational evidence whatever is available) the two approximate theories suggest a straightline path, of somewhat uncertain slope.

Observational evidence is therefore needed to establish

(1) how far downstream the $z \sim x^{\frac{2}{3}}$ law holds;

(2) how significant the intermediate phase is in 'levelling off' the plume;

(3) what the actual plume behaviour is in the final phase.

3. Experimental technique

The smoke plume of the Lakeview Power Station (situated on the shores of Lake Ontario, approximately 11 miles west of Toronto) was the subject of the observations. A single-camera technique was adopted, in which errors of perspective could be accurately compensated for from a knowledge of the relative position of smoke plume and film plane.

The method of data reduction assumed that the instantaneous plume lay in a vertical plane passing through the source in the direction of the mean wind. It would be theoretically possible to determine the instantaneous azimuth angle of each puff of smoke from a bivane trace if the timing were extremely precise. Each puff would then require individual correction. However, a sufficiently accurate approach is to assume each puff to lie in the vertical plane containing the mean wind.

It thus becomes necessary to mount the camera in such a way that the film plane also lies in the vertical plane. This has been accomplished by mounting the camera on a surveyor's transit, thus providing a method for levelling the camera lens and for accurately fixing its azimuth angle. The camera was mounted on the telescope of the transit in such a way that the plane of the camera lens was parallel to the plane of the telescope. The photographic site for each day's experiment was chosen by taking a bearing on the wind direction over a period of 10 min, then driving to a location such that the camera axis could be set up roughly perpendicular to the assumed mean wind direction with a clear view of the entire visible plume. Photographs of the plume were then taken every 2–5 min while the wind data were simultaneously being recorded.

The Lakeview Generating Station operated one or two of its turbine units under full load during all of the experiments with all the flue gases leaving through a single stack. The full load output for each turbine unit was 300 MW. Each experiment was started after 12.00 E.S.T. in order to allow time for the morning inversion layer to be dissipated. In every experiment the data were gathered over a period of approximately 60 min. Chimney height was 493 ft., internal diameter at the top 19 ft. 6 in. The effluent velocity w_0 of the stack gases was calculated from the air flow rate, calculated in turn from the observed feedwater flow rate on the basis of acceptance test data and of the Bailey meter of the power station control room, recording air flow/steam flow ratio. The exit temperature of the flue gases was measured within the chimney 5 ft. below the top.

The smoke plume was photographed many times and the negatives projected on to a screen where the mean plume centre-line could be traced. By superimposing several photographs taken under identical conditions a 'time-mean path' could be found. A minimum of 14 negatives and a maximum of 23 with corresponding periods of photographing of 30 and 58 min were used in calculating any one time-mean plume path. The chimney in the first negative was also traced out, thereby locating the source for each following negative. The co-ordinates of the time-mean path, height above the source versus distance downwind, were then measured on the screen and the corrections for errors of perspective (discussed in detail by Halitsky (1961)) applied.

The horizontal speed of the plume was assumed to be the wind speed at the plume level. This was determined by timing a parcel of the plume over a known distance using a stopwatch. The turbulence intensity in the lateral and vertical directions was recorded at the 2 m level by means of a bivane, along with the wind speed. The intensity divided by the mean wind speed at the same level gave the wind gustiness measurements. As lapse rates were not measured, estimates of stability conditions were based on cloud cover, wind speed and plume behaviour. In neutral conditions and moderate to high winds, the plume could be seen to spread out in a conical fashion from the mouth of the chimney, while in unstable conditions it was often seen to be looping.

Further details of the technique have been described in a report of limited circulation (Slawson 1966).

4. Results

Source and environmental data for the eight experiments are summarized in table 1. As may be seen from table 1, three experiments were carried out during neutral conditions. The observed plume paths in these three experiments are shown in figure 1 in a non-dimensional representation, using $l = F/U^3$ as the scaling length.

A computer analysis of these three plume paths showed that the slope of each plume became constant (within the experimental scatter) beyond a certain non-

Date ...	1. iv. 65	2. iv. 65	6. iv. 65	7. iv. 65	8. iv. 65	10. iv. 65	11. iv. 65	30. iv. 65
Cloud cover (%)	10/10	0/10	10/10	< 6/10	6/10→9/10	< 6/10	10/10	< 0/10
Ceiling (ft.)	20,000	—	7,000	2,500→10,000	2,500→27,000	3,500	5,000	—
Expt. no. and stability	E-1 neutral	E-2 unstable	E-3 neutral	E-4 slightly unstable	E-5 unstable	E-6 slightly unstable	E-7 neutral	E-8 very unstable
Wind speed at plume height U (ft./sec)	33·0	32·0	49·0	27·0	30·0	25·0	38·0	30·0
Gustiness								
Vertical σ_V	0·0930	0·0789	0·0880	0·0929	0·0959	0·1595	0·9873	0·0861
Horizontal σ_H	0·1329	0·2496	0·1348	0·2813	0·2380	0·2346	0·1388	0·3308
Ambient temperature T_A (°R)	490	500	500	517	505	514	500	526
Exit gas temperature T_g (°R)	640	667	686	683	674	678	677	678
Mass flow of gas M_g (lbm/hr × 10^6)	2·04	4·18	4·18	4·13	4·16	4·14	4·19	4·19
Gas exit velocity W_0 (ft./sec)	30·4	65·0	67·0	66·0	66·2	65·4	66·2	66·2
Chimney radius at exit R ft.	9·75	9·75	9·75	9·75	9·75	9·75	9·75	9·75
Buoyant acceleration at chimney b (ft./sec)	9·86	10·75	11·96	10·32	10·78	10·28	11·40	9·32
Flux of buoyancy at chimney F (ft.^{4}/sec^3 × 10^3)	28·5	66·5	70·0	64·7	67·8	64·0	72·0	58·7
Wind speed at 2 m U_{2m} (ft./sec)	24·9	14·5	21·8	14·42	12·9	4·98	17·95	16·25

TABLE 1. Lakeview Generating Station plume rise observations, April 1965

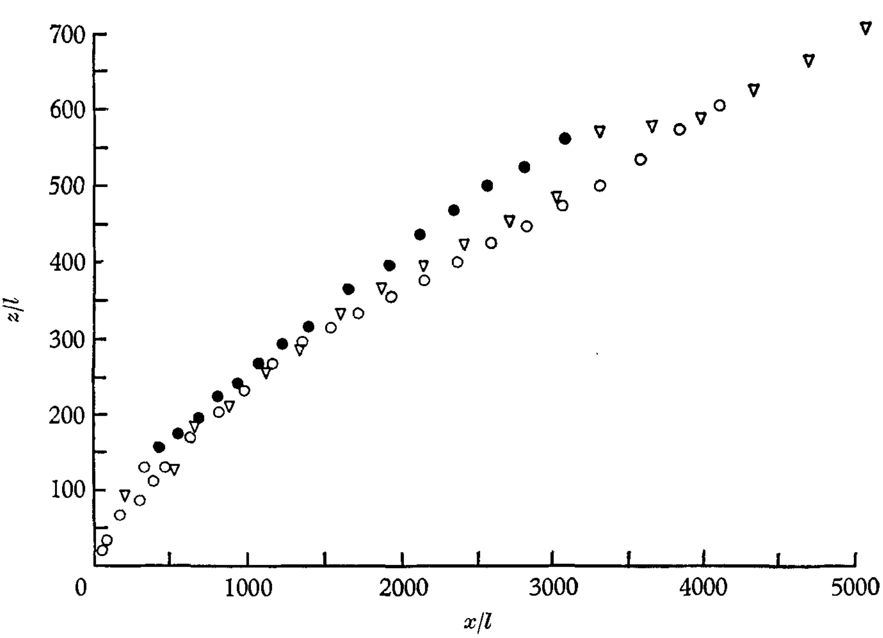

FIGURE 1. Plumes observed under neutral conditions. ○, E-1; △, E-3; ●, E-7.

dimensional distance, x/l, which was approximately the same in each of the three cases, although the slopes were individually different. There was thus a fairly definite transition point (or perhaps a short transition zone) at which the character of the plume changed. Between the chimney and the transition point, moreover, the non-dimensional plot z/l versus x/l of the three plumes coincided almost exactly, giving a 'universal' plume shape in the initial phase. This shape is well described by the equation:

$$z/l = 2\cdot3\,(x/l)^{\frac{2}{3}}, \tag{20}$$

which is of the form of equations (5) and (14). As has been pointed out before, this result follows also from previous experimental data (Csanady 1961, where the constant was given as 2·2), its physical significance being that the dispersion in the initial phase is dominated by the plume's self-generated turbulence.

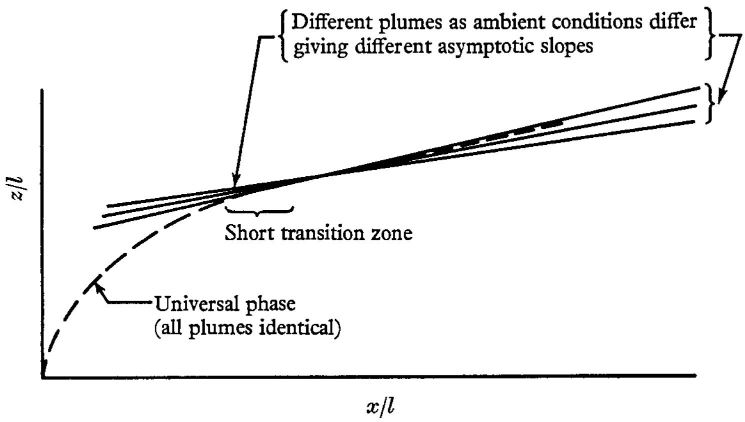

FIGURE 2. Scheme of behaviour of plumes under neutral conditions.

The co-ordinates of the transition point for the three plumes in neutral surroundings are tabulated in table 2. The average co-ordinates are approximately $x_t/l = 1200$, $z_t/l = 280$. This coincides almost exactly with the point where the Tallawarra plume (Csanady 1961) was lost sight of, due to vigorous mixing in a 'break-up' or intermediate phase. It was at that time conjectured that the plume turns horizontal at the asymptotic height given by $z_t/l =$ const., where the constant was very crudely estimated to be 250. It is now seen that, instead of turning horizontal, the plume enters a phase of constant slope. Since individual plume slopes in the final phase are different, presumably because of different atmospheric conditions, the asymptotic slope at Tallawarra may well have been quite low.

The overall behaviour of the three plumes in neutral surroundings is illustrated in figure 2. This sketch more or less answers the three questions posed earlier, at the end of the theoretical discussion. It may be concluded that the 'intermediate phase' of plume behaviour is, at least in this observed instance, insignificant in that it does not lead to any appreciable 'levelling out' of the plume, the asymptotic slope being very much as that of the universal phase-plume at the transition point. The constant plume slope in the final phase agrees with theory.

The numerical value of the asymptotic slope is also given in table 2. It is difficult to compare this with the theoretical expression (19) because the gustiness and scale of turbulence at plume height were not observed. There is, however, some indication that the slope increases with l and is inversely related to gustiness (as measured near ground level), both in accordance with the theoretical result.

Experimental results on the five plumes in a non-neutral (unstable) environment are summarized in figure 3. To explain the relative positions of the plumes in unstable conditions compared to those in neutral conditions one may consider

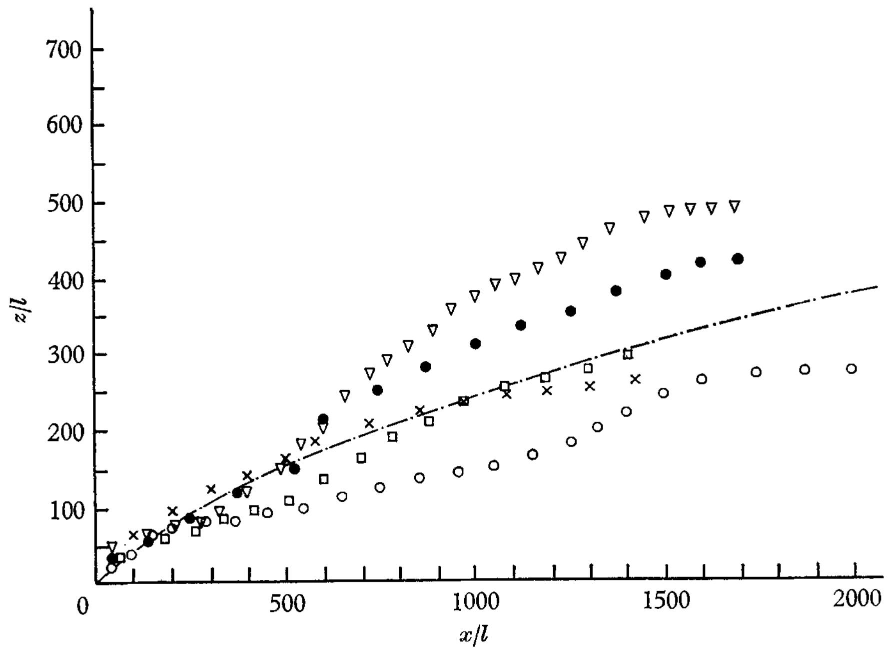

FIGURE 3. Plumes observed under unstable conditions. —·—, plume in neutral surroundings.

Symbol	Expt. no.	l
□	4	3·29
●	5	2·34
×	6	4·10
▽	8	2.17
○	2	2·03

the atmospheric influences of lapse rate and gustiness. These two are opposing influences. (*a*) An increase in gustiness increases the dilution rate of the plume. This increases the plume's effective mass by increased entrainment of surrounding air thus reducing its acceleration and retarding the upward movement. (*b*) As the plume rises its excess temperature becomes greater in unstable conditions because of a reduction in ambient temperature. Thus the buoyant acceleration of the plume becomes greater and hence it will tend to rise higher. Although the gustiness itself depends on the lapse rate, it is also a function of other variables (surface roughness, pressure gradient), so that for the purposes of analysing plume behaviour it may be regarded as a second independent variable.

Referring to figure 3, in those plumes that rise above the plumes in neutral conditions the direct effect of atmospheric instability dominates, more than balancing increased dilution due to increased gustiness. In some other plumes the two effects may balance, while in others the gustiness effect dominates and pulls down the plume. Gustiness values in the unstable experiments ranged from 0·08 to 0·16 in the vertical and 0·23 to 0·33 in the horizontal as against 0·087 to 0·09 and 0·132 to 0·138 under neutral conditions. Either mechanism may apparently dominate the plume rise from time to time resulting in an irregular plume rise pattern.

Experiment	z_t (ft.)	x_t (ft.)	l (ft.)	z_t/l	x_t/l	Asymptotic slope dz/dx
E-1	220	950	0·792	278	1200	0·104
E-3	170	780	0·593	287	1320	0·110
E-7	350	1400	1·31	267	1070	0·140
Average	Not meaningful			277	1200	Not meaningful

TABLE 2. Co-ordinates of transition point for plumes in neutral surroundings

Note that quite close to the source, the plumes in an unstable environment coincide with the ones in neutral surroundings showing that for these also an initial phase dominated by self-generated turbulence exists.

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